Varifocal displays are a practical method to solve vergence-accommodation conflict in near-eye displays for both virtual and augmented reality, but they are reliant on knowing the user’s focal state. One approach for detecting the focal state is to use the link between vergence and accommodation and employ binocular gaze tracking to determine the depth of the fixation point; consequently, the focal depth is also known. In order to ensure the virtual image is in focus, the display must be set to a depth which causes no negative perceptual or physiological effects to the viewer, which indicates error bounds for the calculation of fixation point. I analyze the required gaze tracker accuracy to ensure the display focus is set within the viewer’s depth of field, zone of comfort, and zone of clear single binocular vision. My findings indicate that for the median adult using an augmented reality varifocal display, gaze tracking accuracy must be better than 0.541°. In addition, I discuss eye tracking approaches presented in the literature to determine their ability to meet the specified requirements.

Index Terms: Human-centered computing—Human computer interaction (HCI)—Interaction devices Hardware—Hardware validation—Functional verification Hardware—Hardware validation—Functional verification—Coverage metrics Hardware—Robustness Human-centered computing—Human computer interaction (HCI)—Interaction devices—Displays and imagers

1 INTRODUCTION

In an attempt to solve several of the problems facing Virtual Reality (VR) and Augmented Reality (AR) displays, many recent Near-Eye Display (NED) designs have been presented over the last few years. Among the myriad problems which are being addressed — including field of view, resolution, formfactor, and computational demand — Vergence–Accommodation Conflict (VAC) [13, 14, 19] has been a consideration in almost every recent design. Several classes of display target solving VAC as their primary objective, such as light field displays, multi-plane displays, and varifocal displays. Light field displays address VAC by controlling both the position and angle of the light being emitted. Multi-plane displays address VAC by generating several simultaneous virtual image planes at different focal depths; usually blending them to make a volumetric display. Varifocal displays address VAC by having only one dynamic focal depth image plane which may be set to follow the user’s focus. Of these, varifocal displays add the least optical complexity to traditional NED designs, but have a limitation in necessitating the continuous measurement of the focal state of the user.

1.1 Measuring Focal State

There are two methods for measuring the focal state of a user: the direct method and the indirect method. The direct method works by measuring the light which has passed through the crystalline lens of the eye and bounces off the fundus. Autorefractors image a known illumination pattern in multiple axes and optical powers to determine focal state [6]. Shack–Hartmann wavefront sensors measure the wavefront of light after it has bounced off the fundus to measure the focus of the eye [22]. Both methods are generally performed using infra-red light. Additionally, they require on-axis imaging of the eye, which necessitates more complex optical setups. The fastest running commercial devices available today sample less than 10 times per second. For these reasons, direct measurement of the focal state of the eye is typically foregone in favor of the indirect method.

The indirect method leverages the Human Visual System’s (HVS) coupling of vergence and accommodation [9, 23], and determines focal state by measuring the vergence distance of the eyes. This gaze-based method leverages the more common gaze tracking hardware and, by tracking both eyes, can compute the 3D fixation point, from which the focal depth may also be known. Gaze tracking hardware has several benefits including much higher sampling rates: commonly above 60 hz, but can be as high as 1000 hz, and off-axis tracking capabilities. However, because it does not measure the focal state directly and is based on two parallel systems, any error in determining the vergence has a compounded effect on the error in determining the focal state. It is one of the primary goals of this paper to describe and characterize the effects of gaze-tracking accuracy on focal state accuracy.

It must be noted that if there is dense depth information for the user’s visible environment, an alternative indirect method based on scene depth may be employed. By intersecting the gaze direction with scene geometry, a fast and accurate focal distance may be determined. However this method has a severe limitation in regions near depth discontinuities, in that when the error in gaze tracking accuracy overlaps a discontinuity, the incorrect depth may be displayed. Thus a hybrid methodology will provide the best results; relying on scene depth for good candidate depths and verifying the correct depth by calculating the 3D fixation point.

1.2 Contributions

- A description and characterization of the effects that gaze-tracking error have on calculating the correct focal depth of a user (section 3).
- An analysis of current hardware and techniques for gaze-tracking with specific examination of accuracy to determine how well they may be applied for solving the problem (section 4).

2 BACKGROUND

The foundation of this work is based on the operation of the HVS. Here I describe several areas of interest which will be fundamental in the calculations presented in section 3.

2.1 Panum’s Fusion Area

For a given object depth, there exists a range of eye vergence where stereo fusion will occur. This range is known as Panum’s fusion area (PFA). It is not in the scope of this work to fully describe the topic; the interested reader is directed to the work of Schor et al. [31] and Ogle [25]. Any vergence error larger than 15-30 arcmin will cause a failure in binocular fusion resulting in either suppression or diplopia, while smaller errors will lead to a loss in stereoaucuity [13].

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When presenting virtual stimuli, any inconsistent error in horizontal position of the separate eye’s virtual images will result in incorrect depth perception rather than failure in binocular fusion as the user adapts their vergence to the displayed stimulus. However if unmatched intra-eye distortion occurs beyond the range of DOF, binocular single vision is lost.

2.2 Depth of Field

For a given focal depth, there exists a range of eye accommodations where defocus blur is imperceptible. This region is the depth of field (DOF) or depth of focus. Many studies have been performed in an attempt to characterize the size of the DOF for the human eye and the factors which affect it; the interested reader is referred to Wang and Ciuffreda [37] for a summary.

In the most crude of simplifications, many display makers simplify this to a single value of ±0.3 diopters (d) based on the work of Campbell [2], which works well as a median value from which we can calculate a bounded range based on the circle of confusion.

2.3 Zone of Clear Single Binocular Vision

For any combination of accommodation and vergence depths, there is a zone where binocular fusion and proper focus can occur. This is known as the zone of clear single binocular vision (ZCSBV) [10, 13, 34]. While the size of the ZCSBV varies on an individual basis, generally the boundaries are ±1.5 – 2 d from the natural viewing case.

2.4 Zone of Comfort

While it is possible to fuse images inside the ZCSBV, certain combinations of accommodation and vergence put undue stress on the visual system. Any time spent in these regions will accumulate eye strain leading to visual discomfort and fatigue. However, a subset of the ZCSBV exists where it is safe to remain for extended periods without accumulating strain; this region is called the zone of comfort (ZOC) which is about one-third the size of the ZCSBV [13, 28, 33, 34].

2.5 Accuracy and Speed of Vergence and Accommodation

Under normal conditions, the vergence and accommodation response occur in unison. If a new stimulus is presented at a depth different from the user’s current fixation point, there are three stages of reaction: 1) latency before reaction, 2) initial adjustment, and 3) correction and fine tuning. Interestingly, while the vergence angle adjusts completely to the new depth, the accommodation will lag, settling between the initial focal depth and the stimuli’s focal depth. This is likely due to the strict bounds of PFA driving the vergence, while the looser DOF bounds drive the accommodation [9, 35].

The dynamics of vergence and accommodation, when presented with an unanticipated stimulus, have the following characteristics: vergence latency of 150 – 200 ms and accommodation latency of 300 ms, followed by near equivalent adjustment velocities determined by the magnitude of change, and adjustment periods [3, 16, 24, 32]. However, when voluntary depth change occurs, as with the user-driven fixation in varifocal optical see-through (OST) AR displays, there is no latent period, meaning the expected minimum of 300 ms does not exist [4].

3 Determining Error

3.1 Calculating Distance of Fixation

Most gaze trackers can provide θ azimuth and ϕ altitude data, meaning the 3D gaze vector takes the form

\[ \mathbf{G} = (x, y, z, \theta, \phi) \]

where \(x, y, \) and \(z\) describe the location of the center of the eye. Note that most gaze trackers do not report eye location, so external calibration must be performed, such as simply measuring inter-pupillary distance (IPD), which is typically done for display calibration. Calculating the fixation point is simply finding the intersection of two gaze vectors. Two vectors in 3D space rarely intersect, and while there are several methods of calculating the median of the line of closest approach, the problem at hand is much simpler. Vectors projected onto a plane — assuming they aren’t parallel or diverging — will have an intersection point which can easily be found. The plane intersecting the centers of the eyes and following the head rotation presents a logical choice. With head mounted eye-trackers, the head rotation is already included in the gaze vector and assuming the correct camera coordinate space, the projection is as simple as dropping the vertical position and altitude from the vector and defining our coordinates such that the eye centers lie along the x axis, making

\[ G = (x, z, \theta) \quad , \quad z = 0 \]

the 2d gaze vector equation. Thus it is that only horizontal gaze tracker error and IPD error affect the accuracy of the depth calculation.

Given the form of the vector, trigonometry provides the most straightforward means of converting tracking angle to depth. With the two gaze vectors \(G_L\) and \(G_R\) for left and right eyes respectively, a triangle is generated where the eye center locations form two corners, with angles \(A\) and \(B\) where

\[ A = 90 + \theta_L \]

\[ B = 90 - \theta_R \]

meaning that

\[ C = 180 - A - B = \theta_R - \theta_L \]

with \(C\) being the angle for the triangle at the point of fixation. Having \(c\) which is the IPD, we can use the Law of Sines to solve the other sides

\[ a = \frac{c \cdot \sin A}{\sin C} \sin \theta_L \]

\[ b = \frac{c \cdot \sin B}{\sin C} \sin \theta_R \]

The dynamics of vergence and accommodation, when presented with an unanticipated stimulus, have the following characteristics:
and from Lawes [20] the median of \( c \), or cyclopian distance, \( d \) can be calculated

\[
d = \frac{1}{2} \sqrt{b^2 + a^2} - c^2
\]

\[
= \frac{1}{2} \sqrt{2c^2(\cos^2(\theta_R) + \cos^2(\theta_L))\csc^2(\theta_R - \theta_L) - c^2}
\]

\[
= \frac{1}{2} \sqrt{c^2((\cos(2\theta_R) + \cos(2\theta_L) + 2)\csc^2(\theta_R - \theta_L) - 1)}
\]

(8)

giving us the distance to the fixation point from which accommodation is determined by converting to dioptric distance which gives the focal power \( p \).

\[ p = \frac{1}{d} \]

(9)

A graph for 3 different \( c \) values displays the error conversion for the central field of view in figure 3. Slight variations are seen at different eccentricities, which I leave for future work.

### 3.2 Error Assumptions

Now that we have equations for determining the accommodation, I will examine the different varifocal use cases to determine a specification for each. In order to do so, I must make the assumptions stated here:

- Correct measurement of the user’s IPD has occurred.
- The display is properly calibrated for the user’s IPD and is presenting correct stereoscopic vergence cues for the depth of the stimulus.
- There is proper intra-eye gaze tracker calibration such that the gaze angles from the separate eye trackers are in the same space.
- Seeing as a display should avoid preventing stereo fusion and inability to focus at all costs, I take a tight bound for ZCSBV by using the minimal \( \pm 1.5 \) d.

- As the bounds for ZOC are related to length of exposure, a looser standard can be taken here, which I have done with a value of \( \pm 0.8 \) d.
- While there is some variance in the DOF of a user, I must make a decision based on the wide distribution of reported values [37]. I have selected \( \pm 0.3 \) d.

### 3.3 Differences Between VR and AR

Two common cases for varifocal near eye displays, VR and OST AR, have different requirements when it comes to accurately setting the focal distance of the display. In VR, due to the lack of real world reference, as long as the vergence and accommodation stay within the ZCSBV, the image may be fused and focused. Therefore the display must never instantaneously exceed \( \pm 1.5 \) d of error; however long-term use may lead to discomfort and fatigue. If kept within the ZOC, a user in VR will not experience negative side effects due to VAC, so the long-term error should remain below \( \pm 0.8 \) d. Based on the above equations, this means that in the central field of view the average user requires instantaneous gaze tracking error to remain less than 2.705°, and long-term error to remain less than 1.444° if the display relies wholly on the fixation distance method.

While the zone of comfort also applies to OST AR, due to the real world being visible, the user has an additional reference point, and can directly compare the real and virtual images. When a virtual object is co-located with a real-world object — one of the chief advantages of AR — matching the focus of the virtual to the real can greatly improve perceived image quality. This means that at most, the virtual object focus must be within the DOF of the user; which is assumed to be \( \pm 0.3 \) d for ease of calculation. With this requirement, gaze tracking must maintain an error of less than 0.541°.

### Table 1: Required Gaze Tracking Accuracy in Center Field

<table>
<thead>
<tr>
<th>Dioptric Range</th>
<th>User IPD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 mm</td>
</tr>
<tr>
<td>ZCSBV</td>
<td>( \pm 1.5 ) d</td>
</tr>
<tr>
<td>ZOC</td>
<td>( \pm 0.8 ) d</td>
</tr>
<tr>
<td>DOF</td>
<td>( \pm 0.3 ) d</td>
</tr>
</tbody>
</table>
4 EVALUATION OF NEAR-EYE GAZE TRACKERS

Now that we know how much accuracy is required for different types of varifocal displays, I will review gaze tracking techniques to evaluate their fitness for the task. Many commercial solutions exist, however it is beyond the scope of this work to evaluate them. The Eye Movement Equipment Database, created by Dr. David Wooding of the Applied Vision Research Unit (AVRU) of the University of Derby is now maintained by the Applied Vision Research Centre (AVRC) of Loughborough University and provides links to many manufacturers of eye tracking hardware.

4.1 Intrusive Eye Gaze Trackers

An intrusive eye gaze tracker is any tracker which requires physical contact with the subject while measuring the gaze direction. There are two methods in practice, scleral search coils (SSC) and electrooculography (EOG).

SSC is the gold standard of eye tracking techniques. A coiled wire embedded in a contact lens is suctioned to the scleral region of the eye and the voltage induced in the coil by the surrounding electro-magnetic field is measured [5, 30]. They provide extremely high accuracy of 0.08°, and meet all varifocal requirements, however their dependence for custom and extremely intrusive hardware limits their applicability for widespread deployment.

EOG operates by placing electrodes in the eye region and measuring skin potentials. By detecting differences in the skin potentials, eye motions can be detected and filtered from other signals. It is very common in clinical applications due to its lower cost and less intrusive nature. Accuracies around 2° are reported, so the method is not suitable for varifocal displays [17].

4.2 Non-intrusive Eye Gaze Trackers

Several methods using lights and camera sensors provide methods of gaze tracking that do not require physical contact with the subject.

Infrared oculography (IROG) employs infrared (IR) light-emitting diodes (LEDs) and phototransistors arrayed near the eye. By illuminating the eye using IR LEDs, the phototransistors are able to detect the differences in diffuse reflections between the sclera, iris, and pupil. This results in a voltage difference that is proportional to the angular deviation of the eye [29]. The reported accuracy of 2 min of arc make it a good candidate for varifocal display gaze tracking, and its hardware could be adapted to work with head-mounted displays.

The dual Purkinjie (DP) method developed by Cornsweet and Crane use the first and fourth Purkinjie reflections to separate transnacular head motion from rotation eye motion. Purkinjie reflections are specular reflections from the different surfaces of the eye [7]. The results are an impressive 1 min of arc accuracy, however the complex optical layout and hardware requirements lead to larger formfactors which limit its applicability to varifocal displays.

There is a wide range of video oculography (VOG) techniques which use image features in an attempt to locate the center of the eye in an image, and use a calibration mapping to convert the pupil location to a gaze direction. With names such as Starburst, SET, ExCuSe or ElSe they employ distinct algorithms capable of locating the pupil center in an image [11, 12, 15, 21]. They have reported accuracy of as good as 1°. Having simple hardware could make good candidates for varifocal displays if they achieved better accuracy.

The recent resurgence of machine learning (ML) research has also affected gaze tracking techniques. Balujal et al. and Tew et al. [1, 36] introduced neural networks trained on a combination of near-eye images and synthetic images. Current state of the art results report accuracies in the range of 4.5° [26, 27] and 2.06° [18]. If the accuracy continues trending downward, in the near future ML techniques will have the accuracy required for varifocal display gaze tracking, but for now aren’t ready yet.

4.3 Remote Eye Gaze Trackers

Several techniques and applications exist for remote eye gaze trackers (REGT). Varifocal NEDs is not one of them. The obstruction of the eyes due to wearing the head-mounted display prevents REGT from functioning. Thus I will not review them here.

5 CONCLUSION

Given the perception-based requirements of PFA, DOF, ZCSBV, and ZOC, I calculated the accuracy needed to present varifocal stimuli in different conditions maintaining a natural viewing state. The required accuracy varied based on the IPD of the subject, but for a median user was 1.444° for VR and 0.541° for AR OST.

Unfortunately, many of the current eye tracking technologies found in literature do not have the required gaze accuracy for driving varifocal NEDs. Of those that do have enough accuracy, some require intrusive hardware that make them implausible, while others have complex optical systems which are cost-prohibitive. Of the existing techniques, only IROG meets both the accuracy and hardware requirements for driving varifocal near-eye displays.

It is my aim that future work in eye-tracking will take into account the requirements of driving varifocal near-eye displays, and will target providing a solution which will fit the bill.

ACKNOWLEDGMENTS

REFERENCES


### Table 2: Reported Gaze Tracking Accuracy

<table>
<thead>
<tr>
<th>Method</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scleral Search Coils (SSC)</td>
<td>0.08°</td>
</tr>
<tr>
<td>Electrooculography (EOG)</td>
<td>2.0°</td>
</tr>
<tr>
<td>Infrared Oculography (IROG)</td>
<td>0.033°</td>
</tr>
<tr>
<td>Dual Purkinjie (DP)</td>
<td>0.0166°</td>
</tr>
<tr>
<td>Video Oculography (VOG)</td>
<td>1.0°</td>
</tr>
<tr>
<td>Machine Learning (ML)</td>
<td>2.06°</td>
</tr>
</tbody>
</table>